

Dynamic Constancy as a basis for Perceptual Hierarchies

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RUNNING HEAD: Dynamic Constancy in Perceptual Hierarchies

version: 03-12-07

Abstract: Bateson's difference-based epistemology can be simulated by a Boolean network model. Bateson proposed that taking differences in differences would produce emergent hierarchies of knowledge. This study simulated Bateson's proposal by taking differences in differences in a Boolean model. The crucial result is that constancies in the dynamics of the flow of differences in the model (1) define perceptually comprehensible categories of visual forms and (2) that higher-order constancies arrange these categories into a perceptually comprehensible hierarchy. We propose Dynamic Constancy as a new Gestalt-like organizational principle in perception.

KEYWORDS: perceptual categories; epistemology; Bateson; Boolean networks; Gestalt organization.

A dynamic system is characterized by a flow of changes over time. In a Boolean system these changes are binary, between two values, 0 and 1; they represent what we are calling here a flow of differences. Malloy, Jensen, and Song (2005), have mapped Gregory Bateson's difference-based epistemology onto an NK Boolean system (Kauffman, 1993); particularly relevant here, they proposed a way to represent attractor cycles in a Boolean dynamic system as visual patterns. Bateson, using the map/territory metaphor to distinguish between knowledge and what is known, proposed that what gets onto maps are differences and these differences and their transforms are "elementary ideas" (2000, p. 463). Moreover, Bateson goes on to note that "these differences are themselves to be differentiated" (2000, p. 463). Bateson takes the differentiation of differences even further, making it an important foundation of epistemology by proposing that the process of discovering relationships in the patterns of differences in differences leads to the emergence of a hierarchy of differences with important consequences for mind (see Bateson, 2000, pp. 454-471; 2002, p. 106). Bateson (2000), pp. 463, 464) proposes a mental experiment for the reader:

Let me invite you to a psychological experience, if only to demonstrate the frailty of the human computer. First note that differences in texture are different (a) from differences in color. Now note that differences in size are different (b) from differences in shape. Similarly ratios are different (c) from subtractive differences.

Now let me invite you... to define the differences between "different (a)," "different (b)," and "different (c)" in the above paragraph.

Malloy, Jensen and Song simulated the process of differentiating differences by building an analytic tool, TAO that uses the XOR operator to find the first derivatives of the flow of differences in attractor cycles. To examine Bateson's proposal, above, we extend that logic to higher order derivatives by applying the TAO recursively to the

output of TAO. Thus we recursively take differences in differences. Finally we find constancies in the flow of these differences in differences and examine whether a hierarchy of levels emerges. In this study three realms of description converge on the concept of emergent levels in knowledge: Batesonian epistemology, a Boolean model, and human perceptual experience.

DIFFERENCES IN DIFFERENCES

Higher order derivatives

Clearly, differences are the basis of a Boolean system. An NK Boolean system consists of N nodes each taking inputs from K nodes. At any moment, T , the state of the N nodes is given in a state vector T (e.g., $\mathbf{S}_T = \{0110\dots\}$). As the system changes dynamically from T to $T+1$ to $T+2$, ..., the state vectors, $\mathbf{S}_T, \mathbf{S}_{T+1}, \mathbf{S}_{T+2}\dots$ describe a flow of differences. Malloy, Jensen and Song (2005) described TAO-1, a discrete analogue of the first derivative, which is an analytic tool based on the XOR operator and is used to find differences among differences. In logical terms, the XOR operator returns a 1 if either p is true or q is true (but not if both are true); it returns a 0 if both p and q are true or if both p and q are false. Another way to say this is that XOR detects difference; it returns a 1 if p and q are different and a 0 if they are the same. Suppose, in a 4-node system (see 4-Node Standard in Malloy, Jensen & Song, 2005), we have two state vectors (each showing the state of each of the four nodes in order) $\mathbf{S}_{T=1} = \{1001\}$ and $\mathbf{S}_{T=2} = \{1101\}$. Using XOR to compare each respective position of these two vectors across time, TAO-1 returns a vector of differences in differences (across time) $\mathbf{TAO-1} = \{0100\}$ because from $T=1$ to $T=2$ only the state of the second node is different.

In this paper we explore the utility of extending this logic to TAO-2, TAO-3, and so on (this is analogous to taking higher order derivatives). The point of this extension is

to model Bateson's line of thought that there is a hierarchy of differences in differences in knowledge. TAO-2 is generated by recursively applying the XOR operator to the output vectors of TAO-1. This is the first step in specifying, in terms of a Boolean model, the hierarchy of differences suggested by Bateson. These higher-order derivatives will be described here as general analytic tools for exploring patterns in the flow of changes in N K Boolean systems. These calculations are performed by software called E42.

Insert Table 1 about here

Examine the first column of Table 1 (based on 4-Node Standard, Malloy, Jensen & Song, 2005) which shows the four vectors of an attractor cycle of one basin. (While the term "basin" technically includes both an attractor cycle and the transients (tributaries) that lead into the cycles, we will use attractor cycle and basin interchangeably because the software automatically labels attractor cycles with the name of the basin in which the attractor resides.) Note that these vectors occur one after another deterministically in the dynamics of a small, four-node system and that, since they represent an attractor cycle, that the top state vector {1001} in Table 1 will occur again immediately after the last (bottom) state vector {1011}. To fully code the four vectors of that attractor cycle we would have a matrix whose rows would be the four vectors.

The second column (TAO-1) summarizes the comparisons made by TAO-1 (i.e., by the XOR function) between successive state vectors in column 1. At this point the reader should be able to confirm those vectors. Column 2 lists the TAO-1 vectors: [{0100}, {0010}, {0100}, {0010}] which together could be expressed more properly as a

TAO-1 matrix. Notice that, whereas in column 1 there are four distinct patterns of differences exhibited in the state vectors of basin 1, the pattern of differences in the output of TAO-1 (second column) is less rich, exhibiting only two distinct vectors, {0100} or {0010}, among the four vectors listed. In a similar way to the above process, TAO-2 (column three) is the application of the XOR operator to the output of TAO-1. The XOR operator generates the vectors shown in the third column (TAO-2) by comparing successive vectors listed in the second column and outputting a 1 for each node that is different and a 0 for each node that is the same.

Notice that in the TAO-2 column the pattern of differences is even less rich than it was in the TAO-1 column; in fact there is only one pattern of differences, {0110}, among the four TAO-2 output vectors. The reader should be able to confirm these results directly. Finally, in the fourth column TAO-3 examines the differences among the (TAO-2) differences. Since all the vectors are identical in the TAO-3 column there are no differences in TAO-2 outputs. Therefore all TAO-3 vectors = **0**. At any stage of recursive application, the total TAO ensemble of vectors is, of course, a TAO matrix. Table 1 demonstrates E42's functional operations onto which we are mapping Bateson's verbal descriptions of taking differences in differences. How this leads to a hierarchy of differences is implicit in Table 1 and will be developed below.

Methodologically, it should be noted that Malloy, Jensen and Song (2005) also describe how E42 is able to detect when it is in an attractor cycle, and when it does so to place matrix specifying that attractor in an archive. It then perturbs itself by pseudo-randomly changing the state of fifty percent of its nodes and monitors its own flow until (if) it detects another attractor cycle. E42 continues the previous three steps until it reaches a user-specified number of self-perturbations. Thus it constructs an archive of

attractor cycles, often with hundreds or thousands of attractor matrices. As we will detail below, it then can then take whatever derivative, say TAO-2, and sort all attractors that have the same TAO-2 matrix into a single category. What we find is that at some TAO level, say TAO-2, there will be many categories of attractors; within each category the attractors all have identical derivatives; between categories the derivatives are different. Thus E42 can generate sets of categories at TAO-1, TAO-2, TAO-3, etc. After summarizing how attractor cycles are visualized, we will return to examine how the categories at different TAO levels form a hierarchy.

Visualizing Basins

Rotate the state vectors in Table 1 to be column vectors; then put these column vectors on a grid with 0's represented as white cells and 1's represented as black cells. Figure 1a shows the original state vectors of attractor cycle detailed in Table 1. The system has 4 nodes (ordinate) and sixteen iterations are shown (abscissa). Black cells represent a 1 in a columnar state vector while white cells represent a 0. Since the length of the attractor cycle is four iterations, Figure 1 shows four passes through each cycle. Notice the visual form generated as the behavior of the system unfolds over time. Different attractor cycles generate different visual patterns; Figure 1b shows the form generated by a second attractor cycle in 4-Node Standard.

Insert Figure1 about here

MAPPING PERCEPTUAL EXPERIENCE TO DIFFERENCE IN DIFFERENCES

Hierarchical Perceptual Categories based on Dynamic Constancy

Let us return to the patterns of differences in differences apparent in Table 1 for the recursive application of TAO to the output of TAO. 4-Node Standard gives us a

sense that the variability in the attractor pattern becomes simpler as TAO is recursively applied. But 4-Node Standard is too simple for another crucial insight which is that different attractor patterns can have identical TAO matrices. Since TAO matrices describe the changes in the flow of change, identical TAO matrices means that there are constancies in the dynamics of the flow. How shall we relate these kinds of dynamic constancies, which are a formal characteristic of the model, to Bateson's proposal that an essential characteristic of knowing is a hierarchy of differences in differences? We require a more complex system than 4-Node Standard to explore that mapping fully.

Figure 2 is based on another NK Boolean system with $N=36$ nodes and shows nine attractor cycles (basins) of length $L = 4$ iterations from that system represented as visual patterns. The top row (TAO-4) of Figure 2 shows the 36 nodes on the vertical axis and sixteen iterations (for each attractor) on the horizontal axis; thus each basin pattern of $L=4$ is shown repeating itself 4 times. In Figure 2 these nine attractor patterns are placed into categories in four different ways (bottom row through top row). The rows of Figure 2 are labeled TAO-1, at the bottom, to TAO-4, at the top. It is important to note that Figure 2 shows only representations of attractor cycles; it does not show representations of TAO vectors. While TAO vectors are never themselves represented they are the basis of the sorting of attractor patterns in each of the four rows. One final note on the presentation in Figure 2. Only the top row (TAO-4) of Figure 2 shows the visual patterns for all 36 nodes. The other three rows (TAO's 3, 2, and 1) are reduced; they cut off the top nodes and show only the bottom 21 nodes. The reason is to reduce the size of the figure for printing and to focus on the perceptually interesting parts of the patterns. The full patterns are available to the reader in the top row and it is these nine full patterns that are in fact being sorted in the lower rows. The reduced set of nodes makes perceptual

processing simpler. The top 15 nodes (shown in the TAO-4 row of Figure 2) are necessary to distinguish basins 89, 74, 95 from each other and to distinguish basins 76 and 60 from each other.

Insert Figure 2 about here

In the bottom row of Figure 2 the nine attractor patterns are sorted by TAO-1 matrix identity. For example, the three basins (basins 89, 74, and 95) in TAO-1 category 4 all have identical TAO-1 matrices. Similarly, the two attractor patterns in TAO-1 category 2 (basins 76 and 60) share identical TAO-1 matrices. The rest of the attractors shown in the TAO-1 row are singletons; for the singletons none of the nine basin patterns share the same TAO-1 matrix. The idea is that all attractor cycle patterns that are in the same category share a constant pattern in their flow of change at the level of TAO-1. Another way to say this is that the changes in differences over time are constant for all basin patterns in the same category. We call this Dynamic Constancy and it is proposed here to be a principle for the construction of hierarchies within the model that map to human perceptual experience.

Let us examine higher order dynamic constancies. Examine the second row, TAO-2, in Figure 2. When we use the equality of TAO-2 matrices the patterns are sorted into four categories. This means that the three basin patterns in TAO-2 category 1 (basins 38, 76 and 60) all have identical TAO-2 matrices. Thus basin 38 is now in the same category at the TAO-2 level as are basins 76 and 60, whereas it was in a distinct category at the TAO-1 level. Basin 38 shares second order dynamic constancies with 76 and 60. To emphasize, at each level of this hierarchy the same nine basin patterns are shown, just sorted differently. TAO matrices are nowhere represented; they are the

criteria by which the basin patterns are sorted. Based on identity of TAO-2 matrices, we can see in the TAO-2 row of Figure 2 that the nine patterns can be sorted into four categories.

In contrast to the four categories generated by TAO-2 matrix identity, TAO-3 matrix identity sorts the nine patterns into three categories (second row from top of Figure 2). For example, TAO-3 category 3 is defined by attractor cycles that all have identical TAO-3 matrices (basins 67 and 78). Notice that perceptually basins 67 and 78 are somewhat unlike each other; the characteristic by which they are placed together is a higher level of abstraction. It is metaphorically akin to placing both geese and blue jays in class aves; they do not greatly resemble each other but have abstract characteristics in common. The TAO-4 vector (fourth derivative) is identical for all nine patterns; therefore they are placed in the same category by TAO-4 (see top row of Figure 2). You are asked to look at the hierarchy of categories and check them against your own experience. Note that the same nine patterns are repeated at each level but are categorized differently. Are these perceptually plausible ways of sorting the patterns? If so, at what level? Note that in biological taxonomy a particular person may or may not sort life forms spontaneously as the taxonomy does; the question is after a person knows the taxonomy does the sorting make sense? Many people categorize the forms in categories with the same result as either the TAO-1 or the TAO-2 operations. Even if you didn't categorize them in exactly the same way as E42 did, do the TAO categories appear perceptually plausible? If you did categorize the visual forms differently (from either TAO-1 or TAO-2 or TAO-3) what are the differences? Human visual perceptual experience is taken as the reference point for evaluating the epistemological utility of the model-generated hierarchy in Figures 2. Readers are left to categorize the visual forms

and compare them to the model-generated categories. Leaving readers to their own experience (as opposed, say, to a statistical analysis of performance measures of groups of humans) is deeply motivated by epistemological frames (see Bostic St Clair & Grinder, 2001, p. 71; Malloy, Bostic St Clair, & Grinder, 2005, pp. 105, 113) that place human judgment as the reference point for fields where the patterning being studied is that of humans.

The major results indicate that, first, finding constancies in the flow of differences sorts visual forms into categories and, second, higher-order constancies in the dynamics generate a hierarchy of such categories. As a side note, at a lower level of analysis, the visual forms themselves are generated by finding the dynamic constancy implicit in a system when its flow falls into an attractor cycle.

Boundary Conditions.

The emergence of the kind of perceptual hierarchy seen here is subject to two broad boundary conditions. The first is that such hierarchies exist only for attractor cycle lengths that are powers of two ($L=2, 4, 8, 16, \dots$). If the attractor cycle length (L) is not a power of 2, we find other interesting results. We find categories for non-powers of 2 but they all are at the same level; there are no hierarchies. While the theoretical rationale² for this boundary condition is beyond the scope of this paper and will be addressed in a future paper we can briefly mention that it is due to the fractal nature of the Batesonian process of taking differences in differences.

The second boundary condition derives from the fact that perceptual similarity based on TAO vector identity (dynamic constancy) is an *ceteris paribus* (all things being equal) principle. It is possible to produce examples in which the Gestalt laws of proximity and closure as well as figure-ground relations overwhelm Dynamic Constancy

(TAO identity) in human perception. The reader should be able to derive that the TAO-1 matrix for that subset of three nodes shown in Figure 3 is identical for both basin 89 and basin 57: TAO-1 will yield a series of **1** vectors since every node changes on every iteration in both basins.

Insert Figure 3 about here

Therefore, for these particular three nodes, the TAO-1's are identical but, due to Gestalt organization, these details of the visual patterns do not look perceptually similar as predicted by dynamic constancy; rather they look very different to humans.

Checkerboards and columns are perceptually distinct visual organizations.

Human Judgment, Scientific Description, and Tautologies

Bateson (2002, p.76) defines explanation as mapping a tautology (formal model) onto a scientific description (data, field notes, etc.). In contrast, Keller (2002, p. 5), speaking of biology, grounds explanation as “that which leads biologists to say *Aha!*” Similarly, Gestalt principles of figural organization are confirmed by the direct experience of the observer. Bostic St Clair and Grinder (2001, p. 76) integrate these two types of explanation by proposing that mappings from formal models and instruments to scientific descriptions be validated by human judgment. Examples of human judgment in scientific process include what linguists call intuition, the direct experience evoked in Gestalt examples, and what Keller calls “Aha!” It is in this sense that readers are asked to examine their own judgments concerning our mapping from a formal Boolean model to Bateson's hierarchy of differences. Much like making judgments about a Gestalt law of grouping such as Proximity, your perceptual judgment, whether you agree or disagree,

is the basis for evaluating whether Dynamic Constancy produces perceptually comprehensible categories.

Dynamic Constancy

This study examined Bateson's hypothesis that a key epistemological process is taking differences in differences in differences (refer to his mental experiment above). We have proposed an important addition: Dynamic Constancy. The crucial result is that constancies in the dynamics of the flow of differences in the model (1) define perceptually comprehensible categories of visual forms and (2) that higher-order constancies arrange these categories into a perceptually comprehensible hierarchy. While Dynamic Constancy in this study applies to the sorting of already-organized forms into categories, it is clear that at a lower level the procedure described by Malloy, Jensen and Song (2005) through which individual visual forms emerge is also based on the dynamic constancies implicit in the definition of an attractor cycle.

Because it is based on a formal nonlinear dynamic systems model Dynamic Constancy has a computationally specific advantage over global Gestalt laws in that the process by which it works is specified. Thus the theoretical traction available in nonlinear dynamic systems theory can be brought to bear upon the further study of perceptual hierarchies.

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Table

Table 1			
Recursive application of TAO to Basin 1			
Process	Level-1 Abstraction	Level-2 Abstraction	Level-3 Abstraction
Flow of Differences	Differences in {Differences}	Differences in {Differences in Differences}	Differences in {Differences in Differences }
Original State Vector	TAO-1	TAO-2	TAO-3
T=1	{1001}		
T=2	{1101}	2 vs 1 {0100}	
T=3	{1111}	3 vs 2 {0010}	(3 vs 2) vs (2 vs 1) {0110}
T=4	{1011}	4 vs 3 {0100}	(4 vs 3) vs (3 vs 2) {0110}
		1 vs 4 {0010}	[(4 vs 3) vs (3 vs 2)] vs [(3 vs 2) vs (2 vs 1)] {0000}
			[(1 vs 4) vs (4 vs 3)] vs [(3 vs 2) vs (2 vs 1)] {0000}
			[(2 vs 1) vs (1 vs 4)] vs [(1 vs 4) vs (4 vs 3)] {0000}
		[(3 vs 2) vs (2 vs 1)] vs [(2 vs 1) vs (1 vs 4)] {0000}	

Figure Captions

Figure 1. Sixteen iterations (abscissa) of an $N=4$ node (ordinate) NK Boolean system showing four cycles through an attractor cycle of length = 4 iterations.

Figure 2. Nine attractor cycle patterns sorted into a hierarchy of categories by the recursive application of the TAO operator which takes differences in differences. The TAO-4 rows show all 36 nodes while the other rows show a reduced set of nodes.

Figure 3. Example of a boundary condition showing details from basin patterns 89 and 57. The same three nodes (vertical axis) are shown as they iterate across time (horizontal axis) for both 89 and 57.

Footnotes

2 Joel Cooper and Jonathan Butner contributed in fundamental ways to this theoretical rationale. (editor: refer to page 11 of this manuscript)

Figure 1

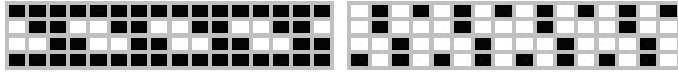


Figure 2

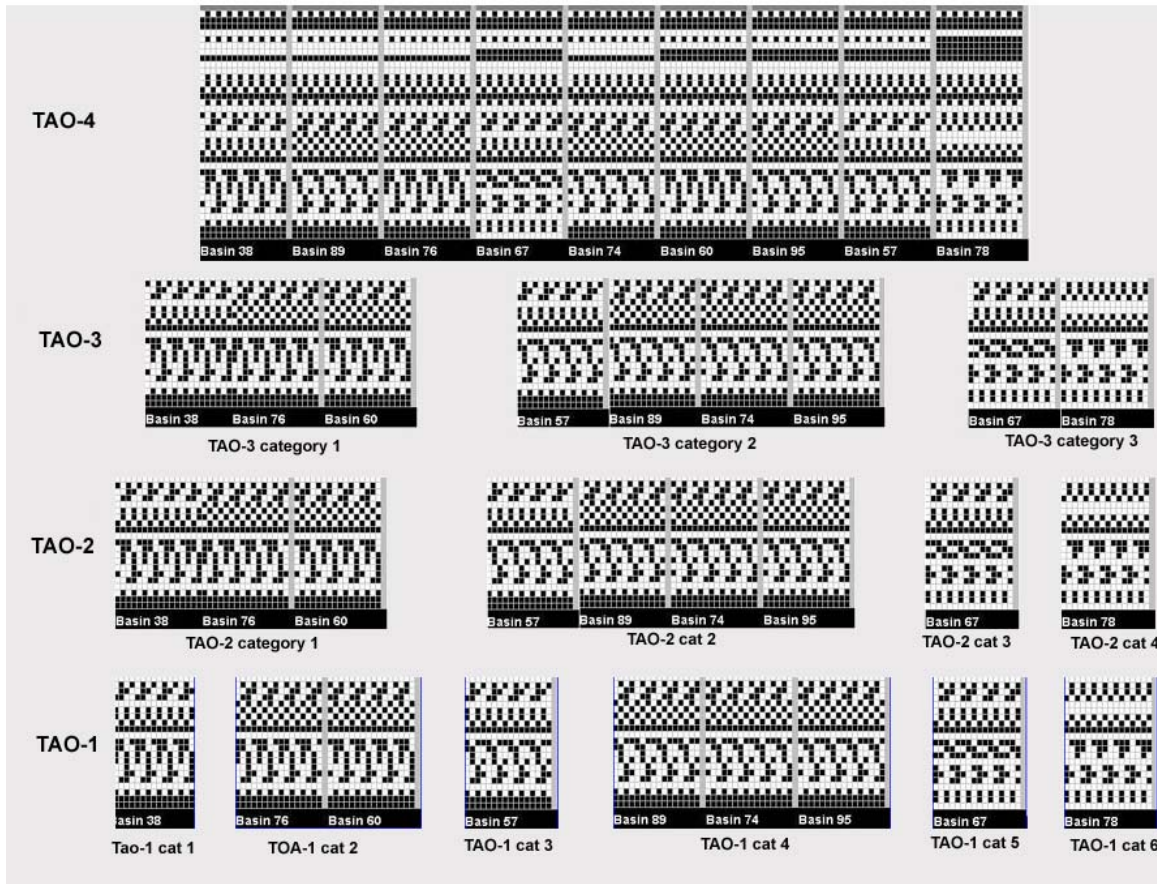


Figure 3

