The emergence of dynamic form through phase relations in dynamic systems

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ABSTRACT

Gregory Bateson construes mental process as the flow and transforms of differences in a system. Stuart Kauffman uses NK Boolean systems to model the emergence of order in biological evolution. Because the Boolean base (0, 1) maps to Bateson's idea of difference, we simulate Bateson's epistemology with a Boolean system. Following Bateson's idea that knowledge emerges from the relations among multiple (at least two) descriptions, where a description is here defined as a systemic process that encodes differences, we propose a perceptual model in which visual form emerges from the phase relations between two such descriptions. The first description is retinal activity, which here is modeled as a discrete dynamic system that falls into different attractor cycles with different fundamental frequencies. Based on the operating characteristics of our simulation model we also propose a second, representational, description. Dynamic form perception emerges from the phase relations between the frequencies of the two descriptions. Moreover, two classes of forms, fundamental and derived, emerge from these phase relations.

Gregory Bateson describes a general epistemology in which mental process is defined as the detection of difference and the transformation of difference as it flows through a network (e.g., Bateson, 2000, pp. 459-461; Bateson, 2002, pp. 85, 86). Stuart Kaufmann (1993, 1995) developed NK Boolean network models to study biological processes in evolution within a dynamic systems perspective. Malloy, Jensen and Song (2005) have specified and extended Bateson's epistemology within a dynamic systems perspective by modeling it within Kauffman's framework using a simulation program, E42, which generates NK Boolean systems. In this paper we will examine how fundamental characteristics of dynamic systems, such as attractor cycles, when mapped Bateson's difference-based epistemology generate a simple model of the perception of dynamic form. Attractors are those aspects of dynamic systems which, like standing waves in a mountain stream, remain dynamically stable in a flow of change. Attractor cycles are strictly defined as derivable characteristics of formal mathematical models but they have been mapped onto scientific phenomena in many ways including the rhythmic motion (gaits) of living beings (Turvey, 1990) and dynamic patterns in general (Kelso, 1995).

The E42 methodology generates complex visual patterns that are both dynamic and systemic. These visual patterns are abstract and unfamiliar, and share the virtues of nonsense syllables and random shapes for being less entwined with past experience than are representations of familiar objects. But the E42-generated visual forms are not primarily an attempt to reduce the effects of familiarity; they derive from a long epistemological tradition (Malloy, Bostic St Clair and Grinder, 2005). Early computation theories (McCulloch & Pitts, 1943; McCulloch, 1965) informed the work of Bateson through his collaborations with McCulloch (e.g., M.C. Bateson, 1991). E42 returns the Batesonian epistemology to the computational realm by integrating it with a dynamic systems approach to the origins of order (Kauffman 1993) in living systems. E42 makes no pretense that it is a replica of the nervous system (neural net models, of which McCulloch and Pitts, 1943, offer an early example, are more applicable for such purposes); rather the purpose of E42's simulation of Bateson's approach is to idealize knowing to its essential process: the flow and transformation of difference in a complex network. For Bateson, a "map" (knowledge system) codes differences which occur in the "territory," whether these be "a difference in altitude, a difference in vegetation, a difference in population structure, difference in surface, or whatever," (e.g., Bateson, 2000, p. 457). "The transforms of a difference traveling in a [system] is an elementary idea" (Bateson, 2000, p. 460).

Since E42 was built to specify and to explore the consequences of the assumption that the flow of difference in a network is the basis of knowing, these forms are produced by a dynamic system whose essential characteristics only exist in the flow of time and some these characteristics cannot be represented statically. Motion and consequently apparent motion is central to this exploration. As Kolers (1972) pointed out "apparent motion speaks to the broad issue of the means by which man represents to himself the characteristics of the world outside his own skin." Imagine a scene. Moving down from above on a steep mountainside carpeted by autumn tinted shrubbery, a herd of tawny deer is obscured by a thick cover of brown autumn leaves, appearing no more than a collection of flickering spots disconnected in space moving together at the same downward angle. How is it that we can extract a coherent pattern that marks the herd of deer as distinct from all the other motion, distinct from the shimmering motion of the leaves in the wind, distinct from the swaying of tree branches, distinct from the motion of a flock of small brown birds flitting among trees along a trajectory that intersects that of the deer, distinct from the motion of the tree trunks which appear to move with respect to the observer who also is moving? There are many motives for extracting dynamic pattern in such a context, curiosity being perhaps the most basic. Whether a being is simply a curious human or bird using information from deer movements to locate berries, whether a being is a predator or a prey, constructing dynamic forms from all the movement that impinges on the retina has fundamental significance. How are coherent patterns formed from this dynamic puzzle? Moreover, a being needs to shift among many distinct dynamic forms. Perhaps a twig snaps provoking a different dynamic form to pop out. The autumn scene is a patchwork of disconnected fragments of motion; first those disconnected fragments that are the deer herd may pop out, then suddenly a flock of birds may pop out, then the leaves of a bush swaying in the wind. What are the processes by which a being shifts from one dynamic pattern, say the deer herd, to another, say swaying seed pods? In this paper we will be particularly interested in the dynamic patterns created by the constantly moving boundaries between the these different areas of motion.

Dynamic systems theory is replete with models that couple two (or more) nonlinear equations, the output of one process acting as the input of others and visa versa. Indeed Turing (1952) addressed the genesis of form through coupled nonlinear equations and Kauffman (1993) uses networks of coupled Boolean nodes to model the emergence of biological order. This is relevant to Bateson's idea of "double description" as the basis of knowledge. He uses "description" in a very general sense; consistent with his usage we will define a description as any systemic process that codes differences. Epistemologically, knowledge emerges through the relationship between two (or more) coupled processes each of which codes difference (2002, p. 61ff, p. 121ff). The generality of the meaning of the term description is indicated by the fact that, for Bateson, depth perception is the result of the relationship between the two descriptions offered by the right and left eye (2002, pp 64-66). Combining these frameworks we will cast form perception as emerging from the phase relationships between two coupled dynamic processes. More specifically, because dynamic systems exist in time and because their attractors are cycles, the kind of framework that comes out of our simulations will suggest that perceived form is a phase relationship between the time flow of retinal dynamics and the time flow of representational processing.

## Method

Malloy, Jensen and Song (2005) have built an NK Boolean simulation program written in Java and named E42 to study how humans know the characteristics of dynamic systems. To use this software, users must enable Java in their web browsers. Moreover, web browsers on PC's must have the Java plugin. Mac's with OS X and using the Safari web browser have Java is built-in; no plugin is necessary.

NK Boolean networks (described in detail by Kauffman 1995, p. 188ff; Kaufmann, 1998, p. 75ff; and Malloy, Jensen, and Song (2005) and extensively at www.psych.utah.edu/dynamic_systems) consist of an arbitrary number N of abstract entities called nodes. Since the model is Boolean, the nodes have two states: ON (state = 1) and OFF (state = 0). Each node takes input (0 or 1) from K nodes. Time flows in discrete iterations. On iteration T, each node, takes the input (0 or 1) it receives from its

K input nodes and uses a logical truth table to determine what its own value will be (either 0 or 1) on the next iteration, T+1. That is, the nodes are coupled; the output of one is input to others and visa versa. As a very simple example, suppose a node, call it Node A, receives input from K=2 other nodes. If Node A is using an AND operator then its state will equal 1 on iteration T+1 only if its two inputs are both equal to 1 on iteration T. Similarly if Node A is using an INCLUSIVE OR operator then its state will equal 1 on iteration T+1 if either its first input or its second input or both are equal to 1 on iteration T. So the relationship between a node's inputs at time T determines its state on the next iteration. This simple model can produce complex dynamic patterns whose changes flow across iterations. NK Boolean systems exhibit the usual characteristics of dynamic systems including attractor cycles (basins) which will be described below and which will play an important role in the perception of the dynamics of such systems.

**State Vectors and Basins**. State vectors, **S**(T), are a convenient way to characterize important characteristics of system dynamics. For purposes of exposition, suppose a system has but four nodes (N = 4). Suppose on iteration T=1 the ON-OFF pattern for the four nodes from first to last is ON, OFF, OFF, OFF. This can be written as a state vector: **S**(1) ={1000}. The sequence of state vectors **S**(1) = {1000} ➔ **S**(2) = {0001} ➔ **S**(3) = {1100} ➔ **S**(4) = {0011} ➔ **S**(5) = {1000} ➔ **S**(6) = {0001} ➔ **S**(7) = {1100} ➔ etc… therefore describes the flow of states (or, alternatively, the behavior) of the system across time. Notice that these state vectors are one dimensional; there is no double index that would generate a 2D matrix. Thus, later, we will discuss these visually as boundaries, more like 1D lines, edges, etc. than like 2D forms or shapes.

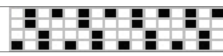
Because the system is deterministic a given state vector, **S**(T), will always be followed by the same state vector **S**(T+1). Therefore the fact that in the above example **S**(5) is identical to **S**(1) means that **S**(6) must be identical **S**(2) and **S**(7) must be identical to **S**(3), and so on. This indicates that the system is in an attractor cycle (in this case of length four) because there are four distinct state vectors, **S**(1) through **S**(4), repeating endlessly. The length of an attractor cycle, measured in the number of state vectors before it repeats, is the fundamental frequency of the cycle. The system cannot escape this attractor cycle unless the system is perturbed. (Perturbation amounts to changing the state of one or more nodes.) A fuller discussion of this example is found in Malloy, Jensen, and Song (2005), where it is named 4-Node Standard, or at http://www.psych.utah.edu/stat/dynamic_systems/. The important points here are that the nodes of NK Boolean dynamic systems flow from state to state by a deterministic relational logic, that at any moment the entire system can be characterized by a state vector, and that the flow from state vector to state vector across time can fall into cyclic attractor basins.

## Static Representations of Dynamics

In order to understand how dynamic temporal forms emerge from the behavior of Boolean systems it is convenient to explain first how system dynamics can be represented statically. This is akin to taking a photo of the action in a team sport. Certain useful relations can be inferred from the frozen spatial relations among the players but the full dynamic pattern of a team playing together is lacking.

**Historical Trace**. Consider the series of state vectors in the preceding discussion. Rotate the state vectors to be column vectors; then put these column vectors on a grid with 0's represented as white cells and 1's represented as black cells. The ordinate will then represent individual nodes from 1 to N and the abscissa will represent iterations from 1 to T. Figure 1a shows the state vectors of the attractor cycle detailed above. The system has 4 nodes (ordinate) whose states vary across sixteen iterations (abscissa). Black cells represent a 1 in a columnar state vector while white cells represent a 0. Notice the visual form generated as the behavior of the system unfolds over time. Different attractor cycles generate different visual patterns; figure 1b shows the form generated by a second attractor cycle in 4-Node Standard. A fuller discussion can be found in Malloy, Jensen, and Song (2005).

**Figure 1**. Sixteen iterations (abscissa) of an N=4 node (ordinate) NK Boolean system showing four cycles through two attractor cycles, a, b, of length = 4 iterations.

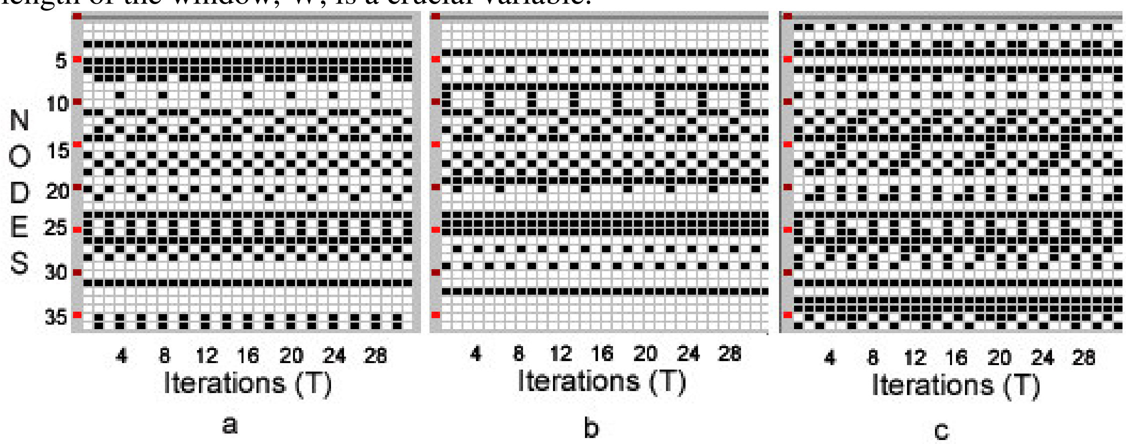
| | |
|---|---|
|  |  |
| **a** | **b** |

**Temporal forms**. Figure 2 shows

Examine Figure 2 which shows four attractor cycles from another small Boolean system. Notice that in Figure 2a that repeated passes through the attractor cycle produces a stripped visual form. In an important early dynamic systems paper Turing in 1952 used coupled dynamics systems to produce patterns akin to animal camouflage patterns as well as other forms in nature. He was speculating about how a genetic code could express itself as form in the actual body of an animal (see Keller, 200x). He called this process morphogenesis, the coming into being of form. In a similar way Figure 2 a through d show the striated attractor patterns of four basins from a small Boolean system that have a certain resemblance to zebra stripes. Based on Kauffman's Boolean idealization of genetic networks and Turing's idea of morphogenesis we can think of the striations in Figure 2 as the developmental growth of hair patterns in an animal's fur. Most important for our discussion is that these patterns are temporal patterns; the hairs grow over time alternatively black then white. Many patterns in nature are essentially of this type. We will think of a boundary of differences (here between black and white) moving across time to produce a temporal form. The boundary itself is 1D, but the resulting temporal pattern is 2D.

As any aside, but related to Bateson's proposal that evolution and knowledge are parallel processes, we can note that Kauffman proposed that evolution proceeds by not just natural selection alone but by the mutual action of natural selection and self-organization. Taking Figure 2 as an oversimplified example of that idea, the genome self organizes into the four basins whose attractors are shown in Figure 2 and an animal can manifest any of the attractor patterns. But natural selection will determine which animals (manifesting which patterns) have survival advantage, perhaps 2a for would have selection advantages in an arid rocky area, 2b for a nocturnal version of the animal and 2c for grasslands. This is a rather simplified description of the main points of one evolutionary approach. Taking Bateson's parallel between evolution and learning, temporal visual forms self-organized in the visual system would be susceptible to selection by utility, reward, social communication and the like.

We now turn, as an example, to a small Boolean system (N = 36 nodes) that has 126 known attractor cycles; the fundamental frequencies of these attractor cycles include 2, 4, 5, 6, 7, 8, 10. Figure 2 shows three screen-captures for three different attractor cycles (panels a, b, and c) generated by this system. As noted above, the abscissa contains the node positions from the first (top) to the thirty-sixth (bottom). White cells correspond to a Boolean 0 and black cells to a 1. For each attractor (a, b, c) the ordinate shows discrete units of time (iterations) from T=1 to T=31. That is, Figure 2a shows a snapshot taken through a window into the dynamics of the system when the system is in one of its attractor cycles. The window is 31 iterations long. Similarly, panels b and c show two other windows, each 31 iterations long, into two other attractors cycles. The length of the window, W, is a crucial variable.

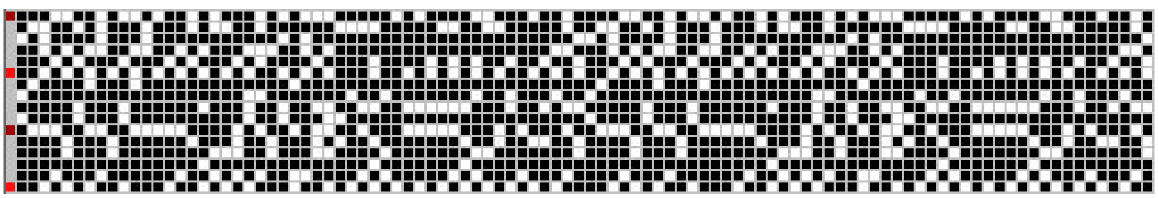



In Figure 2a look at the first (left-most) column. The top cell is white, followed (downward) by another white cell then a black then another white then two blacks and so on to the bottom of the column which ends with a run of five white cells. You can think of the first column as the vertical state vector for the 36 nodes on the first iteration, **S**(1) = {0010110…}. Correspondingly, the second column represents a vertical state vector for the second iteration, S(2) = {0010111…}. Thus Figure 1a shows the flow of state changes of all 36 nodes across 31 iterations. Notice the repetitive nature of the resulting pattern. Within the 31 iterations shown in Figure 1a, the vertical state vectors represented as white and black cells repeat themselves exactly every four iterations; this indicates that the system is in an attractor with a fundamental frequency, L, equal to 4 which is perceivable as a repetitive pattern. The fundamental frequency of the system as it repeats across time is a defining aspect of its form. While we can perceive these characteristics of the system statically, the perception of such patterns in dynamic, rather than static representations, is the central focus of this paper.

Figure 2b shows a different attractor cycle which also has L = 4. Finally, Figure 2c shows the system in a basin with L =7. Translating the flow of state vectors and keeping a visual record of that flow for some number of iterations (in this case 31) is what we call a historical trace; that is, a trace of the system's behavior can be seen through a finite sized window. Figure 2 makes apparent that a historical trace perceptual strategy allows different basins to be easily distinguished from each by a human observer; it is obvious that the system is in different attractors in the three panels of Figure 2. Moreover, if a person is willing to count carefully, the historical trace allows basin length to be determined. As a point of contrast, such perceptual transparency is not characteristic of the Kauffman's perceptual strategy of twinkling nodes (Kauffman, 1993; Malloy, Jensen & Song, 2005).

**Sub-cycles**. Returning to Figure 2a, note that most of the nodes are repeating a pattern of states more frequently than every four iterations (the attractor fundamental frequency). For example, Nodes 1, 2, and 3, at the top of Figure 1a have a sub-cycle frequency of 1, (subL=1); that is, they repeat the same state each iteration. Nodes 15, 17, 18 (among others) repeat the same state every other iteration and so have subL=2. Other nodes in Figure 2a, e.g., 11, 14, 20, repeat every fourth iteration, and account for the system's fundamental frequency of four as a whole system. In other words, the system of nodes as a whole has a fundamental frequency of 4, but individual nodes in isolations may have shorter fundamental frequencies. Similar results can be seen in Figure 2b. In contrast, Figure 2c which shows a basin of L = 7: since 7 is a prime number, the only subL to be found is equal to one (e.g., Nodes 4 and 5). All other nodes repeat their pattern every seven iterations, which is L. The perception of cycles and sub-cycles of dynamic systems is one focus of this paper.

Figure 3 shows a second small dynamic system (N = 20), Exemplar 2. Note that the attractor cycle pattern is more complex; L = 50. Figure 3 shows a window size, W, of 100 iterations (twice through the cycle length). There are no sub-basins, although we will have to wait before we see convincing perceptual evidence of that assertion. As complex as this historical trace is, we will find both more complexity and more simplicity later when we represent this system dynamically rather than statically.



**Dynamic Representations of a System's Dynamics**

**Output Windows**. In Figures 1, 2 and 3 we have presented static images (a snap shot taken through a window) showing a historical trace of the states of the N nodes (abscissa) for some number, W, of iterations (ordinate). To generate dynamic representations of a Boolean dynamic system we will present a series of snapshots sequentially in real time. We will then have what amounts to a real time movie (or more accurately a video) showing the system's dynamics. To create such dynamic representations the E42 program paints a series of static images (windows) to the screen; each image is painted over the previous image, in rapid succession, much like the frames of a movie. E42 tracks the flow of state vectors for W iterations, translates those state vectors into a N x W black and white image similar to the figures above paints it to the screen, and then tracks the next W iterations and paints another N x W array to the screen, and so on. More formally we can say that the screen painting process cycles with frequency W iterations.

**Phase relations**. We have defined two streams of process both of which code streams of differences. To keep with Bateson's abstract terminology, "double description," we call these first description and second description. The first description is flow state vectors, continually calculated in a computer's CPU; it describes the status of system dynamics from moment to moment. In all cases considered here, this flow of state vectors falls into cycles (attractors). The second description is a cyclic process of painting the first flow (of state vectors) to the screen as columns of white and black squares. The second description cycles with frequency W. The phase relations between these two descriptions are critical variables in the following way. When the system is running dynamically, the frequency of the second description, W, has interesting effects that can conveniently be described in terms of apparent motion phenomena. As in the wagon wheel illusion, when W = L, that is, when W equals the fundamental frequency of an attractor, the dynamics appear static (apparent stability) on the screen even though the dynamic system is running. This is because when the window is exactly one attractor cycle length long, each successive image will be identical. Similarly, when W is longer than L, the dynamic patterns appear to move backwards and when W is shorter than L, the dynamic patterns appear to move forward. More abstractly, we can describe the effects of changing the W variable as changing the phase relations between first description (the flow of state vectors) and the second description (painting state vectors to the screen).

While having two descriptions of system dynamics is justified as an operational consequence of Bateson's epistemology, it is also required in the logic of the simulation. In a computer simulation, to represent the flow of a Boolean system's dynamics onscreen we must choose some window size, even if it is W=1. That is, we can paint each iteration as it occurs (W=1), or we can wait two iterations and paint them together (W=2) or wait three iterations and paint those together (W=3) and so on.

## Results

We will now turn to perceptual phenomena that result from manipulating the phase relations between the fundamental frequencies in the Boolean system (L and sub-L) and the frequency W, which is inherent in the process representing the system. Later we will speculate about the possibility that human perception of dynamic form might be modeled in terms of similar phase relations in its perceptual system.

**Adjust frames per second.** If necessary, get the Java plugin. Different computers run at different speeds; what matters perceptually is how many windows (frames) are painted to your screen every second. In all the following applets it is important first to press the "Use Delay" radio button and then to adjust the Delay slider until your particular computer is painting windows to the screen at about 25 to 35 frames per second (fps). The fps readout is to the right of the Stop/Play control bar. The number of fps is a potent variable and you may set it to any value you want by adjusting the delay between each painting of a window to the screen. We suggest the speed be between 25 fps and 35 fps as a useful range for apparent motion phenomena. Also, keep the fps below 65 since most monitors cannot paint accurately beyond about 65 fps. To adjust the delay between each screen-paint you may either drag the Delay Slider or, for finer adjustments, single-click on the Delay Slider Bar either above or below the Slider.

**Perceiving fundamental dynamics**. The Exemplar 1 applet allows the user to adjust W variable in relation to attractor cycle length, L, and sub-cycle length, subL. Consistent with apparent motion effects, when the frequency of W equals fundamental frequencies (L or subL) the dynamic pattern (or part of it) freezes. The default basin of Exemplar 1 has L=4, while the applet defaults to W=69 which is not divisible by 4, so you will see apparent motion upon pressing Play. Only nodes that have sub-cycle length

= 3 (or 1) will appear apparently stable. You may drag the Window Size Slider to change W to change phase relations between the Boolean dynamics (first descriptions) and the screen paint process (second description). To get small increments or decrements in W you may click on the Slider Bar above or below the Slider. When you change W to 72 the whole display stops; the system is still running and what you see is apparent stability. When you change W to 71 the whole basin moves as a coherent whole. When you set W to 70 only those nodes with subL=2 will exhibit apparent stability. As noted above, setting W to any multiple of 3 results in apparent stability for those nodes with subL=3.

To change attractor cycles, you can perturb the system by clicking the perturb button near the bottom of the control panel; this changes the values (from 0 to 1 or visa versa) of some fifty percent of the nodes. Typically, but not always, this will provoke the system into a different attractor basin. If the system does not change attractors, click Perturb again. While all the particulars change for the new dynamic pattern resulting from a new attractor cycle, the generalizations noted above remain verifiable.

This applet demonstrates perceptually that shifting the phase relations between the fundamental system frequencies (L, subL) of the first description and the frequency of the second description produces changes in dynamic forms that highlight various fundamental characteristics of the system either by making them static or by changing their movement pattern. In other words a simple shift in the phase relations between intrinsic systemic dynamic characteristics and what might be called the perceptual process of screen painting can highlight fundamental characteristics of dynamic behavior by either freezing them in apparent stability or making them apparently move differently than the rest of the system.

**Perceiving derivative dynamics**. We now examine how the phase relations between the two descriptions generate forms that are not fundamental to the system but are solely derivative of the interaction between the two descriptions. Let us look at a second example and then discuss this further. The Exemplar 2 applet demonstrates that system characteristics that are not fundamental frequencies of attractor cycles can pop out perceptually through manipulating the phase relations between the first description of system process (flow of state vectors) and the second description of system process (screen paint), using W as a control variable.

Figure 3 shows a static version of the default basin for the Exemplar 2 applet; this system has two other known basins. Press Play and adjust the Delay so that fps is between 25 and 35. The applet loads with W = 75; at that setting islands of stability (that are not present in the static image of Figure 3) appear in the midst of a grey background (black and white cells alternating quickly). These islands are neither basins nor sub-basins. They are visual forms that have apparent stability and that emerge from the phase relations between the first and second descriptions. If these phase relations are changed, if W is set to 74 or 76 (click below or above the Slider on the slider bar), these islands will remain as coherent units but begin moving. Further changes in the relationship between first and second descriptions produce more complex perceptual results: At W = 77 other apparent forms emerge. These emergent forms are not found in static images of the system (e.g., Figure 3 nor are they found when the Stop button is pressed on the applet). The fundamental conceptual point is that the emergence of these dynamic patterns is a co-construction of two descriptions of the system's dynamic behavior.

**Ambiguous motion**. At W = 77, ambiguous motion also appears. Ambiguous motion can be observed for Node 5 (fifth node from the top, lined up with a red hash mark). Move a pointer (the mouse arrow or a pen tip) horizontally along Node 5 from left to right and then right to left; the black squares will move with the direction of your pointer. (The user may have to change the horizontal speed of movement of the pointer to obtain this perceptual effect.) With a little practice users can provoke this change of direction of movement with their eyes alone. With W = 83, complex emergent forms with ambiguous motion can be perceived moving either right to left or left to right; once again, you may require a horizontally moving pointer to observe this phenomenon.

Since the fundamental frequency of the default attractor cycle is L = 50, integer multiples of 50 with produce a static pattern that looks like Figure 3, even thought system is running.

As with other apparent motion phenomena, the speed of representation in frames (windows) per second also affects perception and can be explored by manipulating the Delay Slider. Results at 50 fps can be strikingly different than at 25 fps.

Finally, explorations with Exemplar 3 will reveal similar phenomena to those found in Exemplar 2 but will have a resemblance to wave patterns like those in the flat area between a beach and the surf. At the default W = 91 and with the speed set to about 35 frames per second an ephemeral wave pattern moves from right to left while a choppy and more robust wave pattern moves more or less from left to right. Other window sizes produce strikingly different dynamic patterns. Once again, these patterns don't exist in the fundamental frequencies of the system itself but emerge in the phase relationships between the two descriptions of the system's dynamics.

## Discussion

**Summary**. In the Exemplar 1 results we see that manipulating phase relations between a first and second description of system dynamics produced visual forms that reflected fundamental characteristics of the system (L and subL). Notice also that in these cases, consistent with Bateson's general epistemological principle of double description, such dynamic forms do not exist in either description alone but emerge in the relations between the two descriptions.

In Exemplars 2 and 3 we see the emergence of dynamic forms that are not fundamentals of the system's attractor cycles but are solely derivative of the phase relations between descriptions of the systems' attractor cycles. As we will discus below this is intriguingly parallel to the idea that sentient beings, particularly mammals, primates, and humans don't simply extract "that which exists" in the environment but rather that their perceptual process interacts with "that which exists" in ways that actively co-construct or "enact" their experience in ways that go beyond the information given.

As we noted above, in a simulation we need two processes (descriptions) to represent the Boolean system on the screen; we need a computational process in the CPU and a second process to represent the computations on the screen. Granted that is so, some choice must be made about their phase relations; this choice implies a phase relation between the system's dynamics and how those dynamics get represented. If, as we do below, we assume that this in some way models human perception of systemic dynamics, then specifying phase relations between aspects of the to-be-perceived dynamics and aspects of representational processing is a potentially crucial variable in perceptual experience.

**Mapping the Model to the Retina**. The demonstration of the consequences of manipulating phase relations between the flow of E42's computational dynamics in a computer's CPU on one the hand and the screen outputs of E42 on the other hand is one kind of endeavor. The mapping of these consequences to human perception is another, more hazardous and speculative, endeavor. Many mappings are possible; one will be proposed here in the spirit of starting a discussion. For the moment assume that what the dynamic system E42 is modeling is the retinal image. Before we pursue that model, we will be specific about what we are not doing. We don't propose that E42 models the any aspect of the universe such as the external ambient optical array. Nor are we proposing to build replicas of actual neural activity and pathways in the retina (neural nets and other models do that better). Rather, we are modeling Bateson's proposal that what gets onto maps from the territory are differences and his proposal that, of its nature, knowledge is a flow of transformed differences in a network; thus we suggest that E42 act as an (extremely) idealized and abstracted model of the retina's response to the universe in the Batesonian difference-based sense. We are also modeling Bateson's proposal that knowledge emerges from the relations among multiple descriptions defined as multiple flows of difference.

While we don't propose to model the territory, we do assume that the retina, construed here to be a discrete dynamic system, is coupled (entrained) to the processes of the ambient environment (e.g., Turvey, 1990, p. 942). Therefore we assume that the dynamics of the retina, particularly the basins into which if falls, have a useful relationship to the dynamics of the territory. Given, then, that the retinal image is (modeled as) a discrete dynamic system coupled to the environment, we propose a very simple of model of the emergence of dynamic visual form: Form emerges through phase relations between at least two streams of differences flowing in a richly connected network. Following Bateson's framework we call these two streams of process descriptions. One stream of process, the retinal image, we will call the first description; the second stream (representation to the screen in the simulations) we call the second description. Description may seem to be a strange word for visual phenomena; in using it we are following Bateson's general usage that includes visualizations. Thus we propose that form emerges from phase relations between flow of the first (retinal) and second (representational) descriptions. We add another assumption: The perceptual system has some mechanism for adjusting the phase relations between these two descriptions. This adjustment allows the popping out of different dynamic forms. In terms of human development and perceptual learning, the ability to shift and stabilize these phase relations would depend on the utility of the emergent forms for a particular being relative to specific contexts. Humans would learn, for pragmatic and cultural reasons, to perceive certain forms by making adjustments in the frequency to the second description (representational process).

**Fundamental forms versus derivative forms.** The model, simple as it is, makes a important distinction between emergent forms based on the fundamental frequencies of retinal dynamics (Exemplar 1) on the one hand and emergent forms derived solely from the phase relations between the two descriptions (Exemplars 2 and 3). The first case we will call fundamental forms and the second, derivative forms. In either case, fundamental or derived forms, the forms emerge from the phase relations between retinal and representational process. But the fundamental forms are arguably characteristics of retinal dynamics; they are visualizations of its attractor cycles and sub-cycles. The derived forms emerge solely from the relations among the two descriptions; this does not mean that they cannot be stable and useful. Given the retinal image is entrained with a particular environmental context whose dynamics are stable across time, these derived forms would then emerge reliably for each visit to that context.

**Conclusion**. The retina is construed to be a dynamic system that is coupled to the environment; the nature of retina-environmental coupling is not modeled here. But assuming such a coupling exists, the retina is modeled as an NK Boolean system that has characteristics typical of dynamic systems in general such as attractor cycles. Presumably these systemic characteristics are related to dynamically systemic characteristics in the environment. Bateson proposes that useful knowledge requires, as minimum, a double description. In that framework we have described two descriptions (retinal process and representational process) whose phase relations allow the perceiver to pop out forms that represent fundamental systemic frequencies (attractors and sub-attractors) of the retina. Such fundamental forms are proposed to be the theoretical basis for aspects of perception that have high consensus across observers. In contrast the phase relations between these two descriptions can also pop out forms that are not systemic characteristics of retinal dynamics; they solely derived from the phase relations between retinal dynamics and perceptual dynamics. Such derived forms might be the theoretical basis for aspects of perception that are more subjective, such as forms people perceive in clouds that may not be immediately apparent to other people but can be perceived after some sort of perceptual adjustment. In terms of the autumn mountainside scene, systemic characteristics (Exemplar 1) might map to the perceptual form corresponding to a whole deer running in the open with no visual obstruction, while emergent forms (see particularly Exemplar 2) might map to the group of brownish blobs moving in unison through the trembling leaves of bushes; these moving blobs in no way are aspects of deer per se but can have high utility in hunting deer. These blobs are not fundamentally the form of deer but are forms derived from the phase relations between retinal and representational processing.

This is both a simple and a dynamic model for complex, dynamic form perception.

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**Figure Captions**