

HOMEWORK: NORMAL PROBABILITY DISTRIBUTION

Go to the StatCenter Main Menu. From there click on “Fun” and then click on “Probably Fun.” Under “Probably Fun” click on “Normal Distribution.”

At the very bottom of the Normal Distribution Menu, you will see a button for “Probability Tool for ANY Normal Distribution.” Use this tool to answer the first two questions below. Later, you will be asked to return to this menu and click on the second button from the top (“Probability Tool for the UNIT Normal”).

Please note that none of the three story problem buttons on the Probably Fun menu leads to anything but JAVA code. On this paper you are reading is our first set of story problems. After we have tried these story problems out, we will put them on the web. For now, read the story problems off this paper and then use the Probability Tools to answer the questions. Turn in your answers (on paper) to the TF’s.

We use different symbols for the MEAN and STANDARD DEVIATION when we refer to probability distributions (rather than samples consisting of a set of numbers). The mean of a probability distribution is symbolized by the Greek letter “mu,” (μ). The standard deviation of a probability distribution is symbolized by the Greek letter “sigma,” (σ). The variance of a probability distribution is symbolized by “sigma squared,” (σ^2).

For every problem below, sketch (by hand) a small normal distribution which shows its mu and sigma. On this sketch show the relevant scores (e.g., upper and lower scores) and shade in the area that is asked for by the problem. These problems are next to impossible to understand unless you VISUALIZE the relationships asked for. So the combination of seeing the computer draw the picture and then drawing the picture yourself is an important part of learning to think in a way that makes these kinds of concepts possible and even easy. The computer is directing your attention toward learning a “cognitive strategy” which allows you to translate the verbal/conceptual material in the story problem into the kind of visual relations that allow you to think successfully about this material.

1. A researcher develops a standardized test of Spatial Ability. Based on a national sample of thousands of research participants, he concludes that his test of spatial ability (TSA) is distributed normally with $\mu = 150$ and $\sigma = 30$. That is, the theoretical population (probability distribution) of TSA scores has a mean of

150 and a standard deviation of 30.

A) Use the probability tool to find the probability that a randomly selected person will score between 110 and 160 on the Test of Spatial Ability (TSA).

B) Use the probability tool to find the probability of a TSA score either below 110 or above 160. That is, what is the probability of finding a TSA score “outside” of 110 and 160?

C) Use the probability tool to find the probability of a TSA score below 160. (Hint: Use the minus infinity setting for the lower score.)

D) Use the probability tool to find the probability of a TSA score above 110. (Hint: Use the positive infinity setting for the upper score.)

E) Use the probability tool to find the probability of a TSA score between negative and positive infinity.

F) What is the probability of a TSA score between 120 and 180?

G) What is the probability of a TSA score between minus and plus one standard deviation (F)?

2. Continue with the example from problem (1) above. We are now going to examine a series of questions whose answers will make the future learning of certain statistical concepts much easier. For now, just answer the questions. Your experience with generating the answers will help you later.

A) What is the probability of a TSA score above 209.

B) What is the z score of the raw score 209. [$z = (X - \mu) / \sigma$]

C) What is the probability of a TSA score below 91?

D) What is the z score of a TSA score of 91?

E) What is the probability of a TSA score between 91 and 209?

F) What is the probability of a TSA score outside 91 and 209?

F) What is the probability of a TSA score outside -1.967 and +1.967 standard

deviations (sigmas)?

Click on the “Back to Menu” button at the bottom of the probability tool screen. You will see the Normal Probability menu. The second button from the top is the “Probability Tool for Unit Normal (z).” Press this button to use the Unit Normal Tool.

The Unit Normal distribution has a mean (μ) = 0 and a standard deviation (σ) = 1. Any normal distribution can be converted to the unit normal by turning all scores into z scores. For a probability distribution $z = (X - \mu) / \sigma$. The Unit Normal is frequently put in tables in the back of statistics books.

When all scores are expressed as z scores, you have the unit normal. So you will notice that the Unit Normal Tool asks for z scores as inputs.

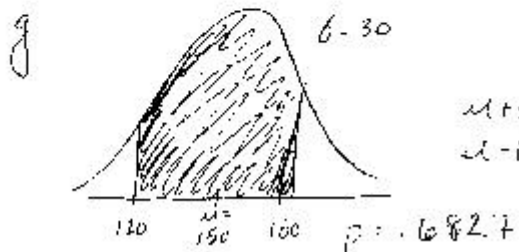
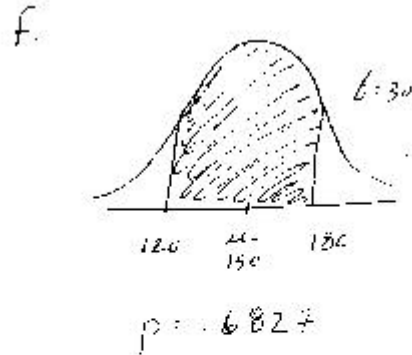
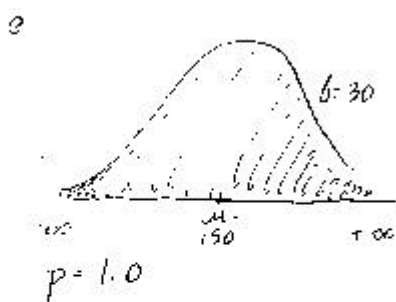
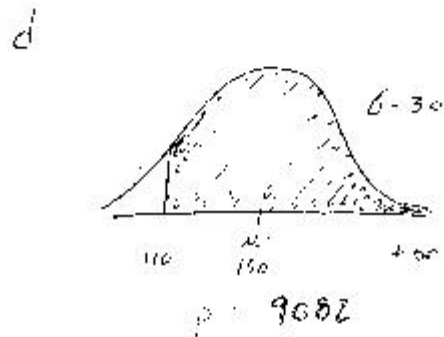
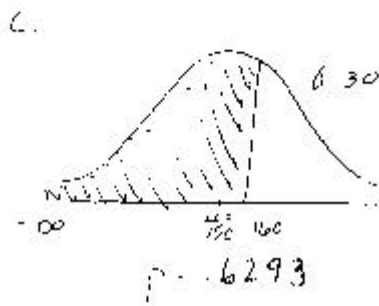
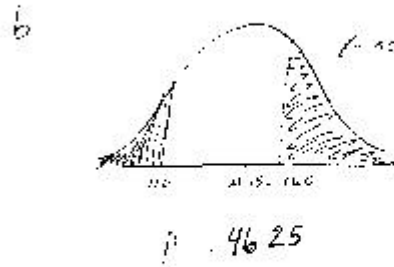
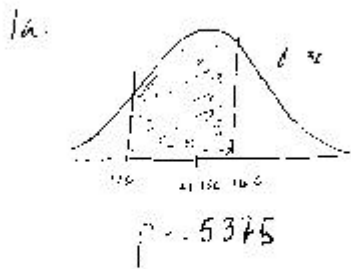
Remember to continue to sketch a probability distribution, shading relevant areas, for each answer.

3. A) Using the “Unit Normal” probability tool find the probability between -1 and +1 standard deviation from the mean. (Hint: What is the z score of -1 standard deviation?)
- B) Above (outside) what z score does .05 of the probability lie? (Hint: Dragging the blue upper (or lower) score pointers is the easiest way to use the tool to answer this question.) (Answer: Depending on how you like to round, you will get either +1.64 or +1.65)
- C) Below (outside) what z score does .05 of the probability lie? How much of the probability lies above that z score?
- D) Above what z score does .025 of the probability lie? How much of the probability lies below that z score?
- E) Below what z score does .025 of the probability lie? How much of the probability lies above that z score?
- F) Now combine z scores from the previous two answers. How much of the probability lies BETWEEN those two z scores? How much probability lies OUTSIDE those two z scores?

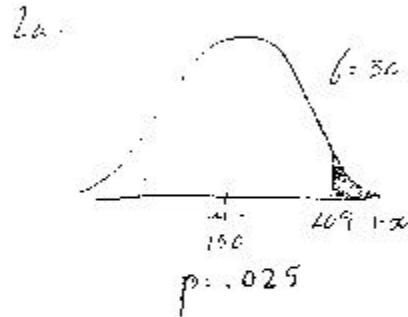
Problem 3 should be straight-forward. Make sure you understand it. The z scores you

found, and especially the pictures you drew, for problems B through F above are very important in understanding the logic of topics we will cover in inferential statistics (rejection regions, one- versus two-tailed tests, etc.) For example B, C, and F represent three ways to get one or two areas containing a probability of .05. For now, just notice these three ways to get an area containing .05.

Normal Distribution



$\mu + 1\sigma = 150 + 30 = 180$ upper
 $\mu - 1\sigma = 150 - 30 = 120$ lower

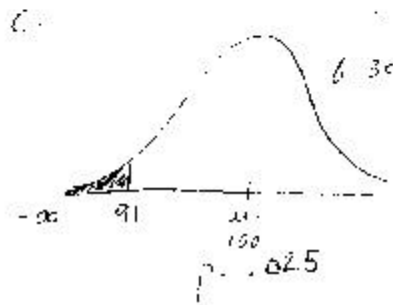


b.

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{209 - 150}{30}$$

$$= 1.967$$

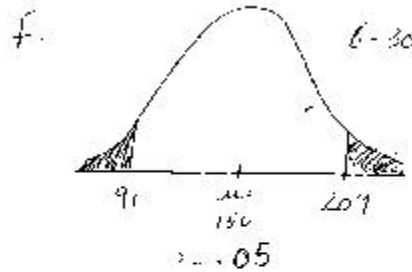
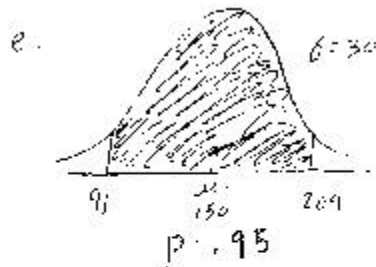


d.

$$z = \frac{x - \mu}{\sigma}$$

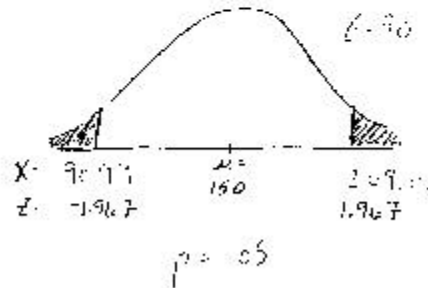
$$= \frac{91 - 150}{30}$$

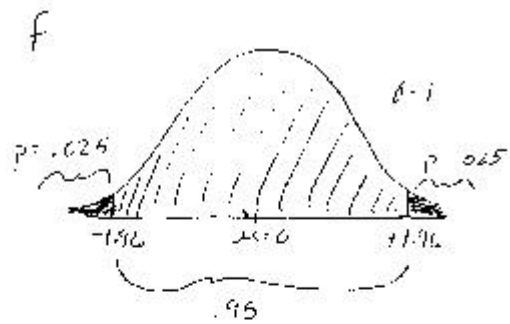
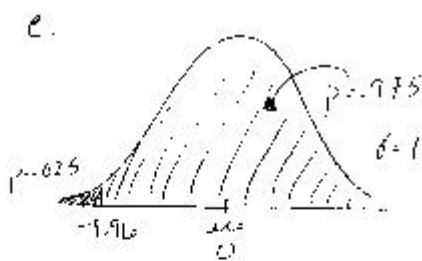
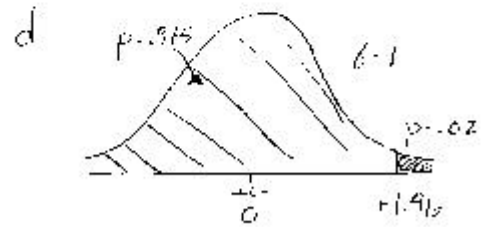
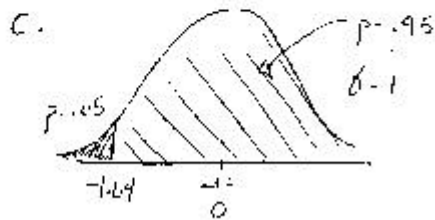
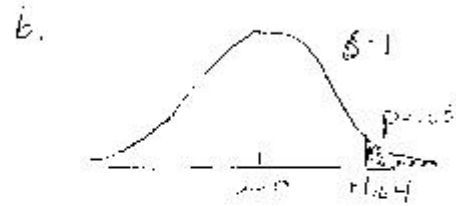
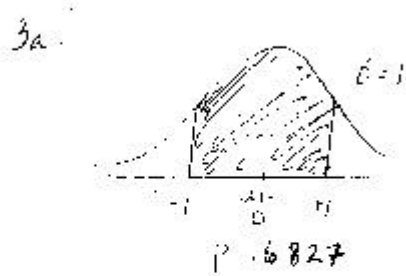
$$= -1.967$$



g. $z = \frac{x - \mu}{\sigma}$

h. $x = \mu + z\sigma$
 If $z = -1.967$
 $= 150 + (30)(-1.967)$
 $= 90.99$
 If $z = 1.967$
 $x = 150 + (30)(1.967)$
 $x = 209.01$





outside = .05

