HOMEWORK: BINOMIAL PROBABILITY DISTRIBUTION

Go to the StatCenter Main Menu. From there click on “Fun” and then click on “Probably Fun.” Under “Probably Fun” click on “Binomial Tool.”

Read the following story problems and then use the Binomial Probability Tools to answer the questions. Turn in your answers (on paper) to the TF’s.

The Binomial Probability Distribution can be used to model simple processes that have two possible outcomes such as flipping a coin. To be concrete I will use coin flipping to describe the parameters of the binomial distribution. When you flip a coin there are two possible outcomes: Head and Tail. Arbitrarily one of these is called a “success” and the other a “failure.” In conversations about the binomial, success and failure do not have their usual connotations; they are simply used to distinguish one outcome from another. In coin flipping examples, we usually call a “head” a “success.”

The binomial distribution has three parameters: N, r, and p. For coins, N represents the number of times you flip the coin (or the number of coins you flip). r represents the number of successes you get. So if you flip one coin r can be either 0 or 1, because you will either get zero heads or one head. But if N = 2 (you flip the coin twice), then r could be 0 or 1 or 2. And so on. If you flip the coin 20 times (N = 20), then r can vary from 0 to 20. p represents the probability of a success (getting a head). For fair coins, p = .5. But we can imagine (at least conceptually) coins that have been tampered with so that the probability of a head (p) could have any value between 0 and 1. So when using the binomial distribution, p does not have to be .5. q = 1 - p and represents the probability of a failure.

We don’t use these much in the binomial distribution, but for your information the mean (:) of the binomial = Np. The standard deviation (F) of the binomial = square root (Npq).

For every problem below, sketch (by hand) a small binomial distribution which shows the relevant scores (e.g., upper and lower values of r) and shade in the area that is asked for by the problem. As with the normal distribution these problems are next to impossible to understand unless you VISUALIZE the relationships asked for. So the combination of seeing the computer draw the picture and then drawing the picture yourself is an important part of learning to think in a way that makes these kinds of concepts possible and even easy. The computer is directing your attention toward learning a "cognitive strategy" which allows you to translate the verbal/conceptual material in the story problem into the kind of visual relations that allow you to think successfully about this material.

1. An instructor develops a True-False exam for an introductory stat course. A student, who didn't study, randomly guesses on each question. Making the reasonable assumption that the probability of success (guessing correctly) on each question is equal to .5, answer the following questions.

   A) If the exam has 20 questions, use the probability tool to find the probability that the student will guess exactly 5 right. Exactly 11 right. Exactly 19 right.

   B) If there are 20 questions, what is the probability that the student will make BETWEEN 7 and 12 right guess? [In the binomial problems we will take the convention that "between" is inclusive. That is, when I say "between 7 and 12" correct, I mean to include 7 and 12.]

   C) What is the probability that the student will score outside of 7 and 12 correct? [Here we will take the convention that "outside" is exclusive, that is, "outside 7 and 12" does not include 7 and 12.]

   D) What is the probability that the student will score 18 or above?

   E) What is the probability that the student will score 14 or above?

   F) What single score is the most likely? And what is the probability of obtaining this score?

   G) If there is only one question (N = 1), what is the probability of getting 100% by guessing?

   H) If there are two questions, what is the probability of getting 100%?

   I) If there are five questions, what is the probability of getting 100%?

   J) If there are 10 questions, what is the probability of getting 100%?

2. At the State Fair there is a booth where people can throw dimes onto a table that

has dishes on it. Suppose that the chance that a dime lands on a dish is equal to .3, that is, p = .3. Suppose for a dollar you can buy 10 throws.

A) The most desirable prizes (on the highest shelf) require that you get 9 or 10 (out of 10 dimes) in dishes. What is the probability of getting 9 or 10 out of 10?

B) The second highest requires that you get 7 or 8 dimes in dishes. What are your chances of doing that?

C) The third highest shelf requires that you get either 5 or 6 dimes in dishes. What are your chances of doing that?

3. When N is large (say > 100) the binomial distribution begins to approach the normal distribution in shape. This is especially true when p is around .5.

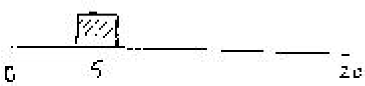
A) Set N = 100, p = .5. Describe the shape of the binomial distribution.

B) Set N to some number larger than 100 (but less than or equal to 300, since the program only allows N up to 300). Leave p = .5. Describe the shape of the binomial.

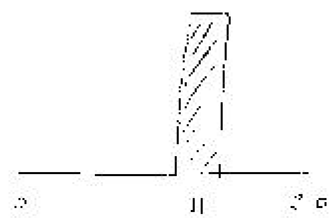
C) Play with leaving N high, but changing p from .5 to .4 then to .3 then .2 then .1 then .05 and then even as small as .01. What do you discover?

Binomial Probability Distribution

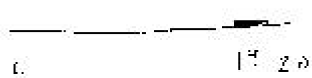
1.a



p = .0148

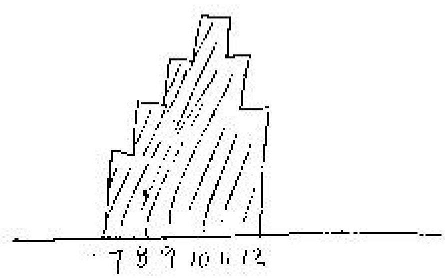


p = .1602



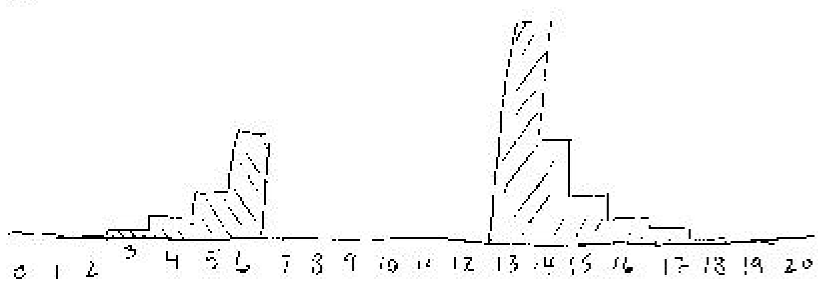
p ≈ 0

b.



p = .8108

c.

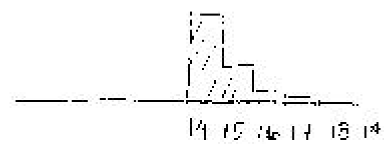


p = .1892

d.

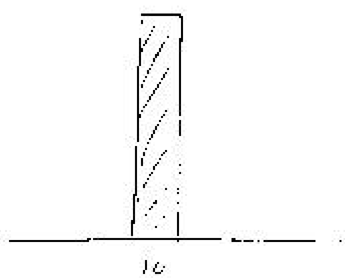
p = .0002 (2.0E-4)

e.



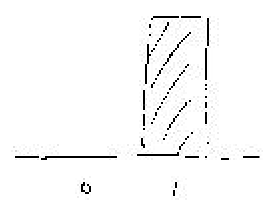
p = .0577

f



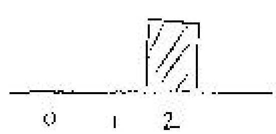
p = .1762

g



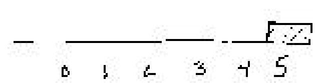
p = .5

h



p = .25

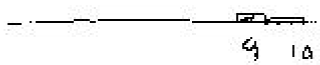
i



p = .0313

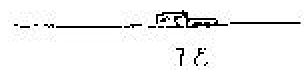
p = .001

2. $p = .3 \quad N = 10$



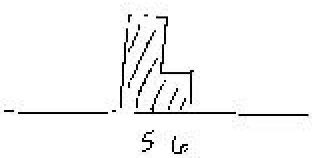
$p = .0001$ (1.0 E-4)

b.



$p = .0104$

c.



$p = .1397$

3. $N = 100$, $p = .5$
Starts to look more like a normal distribution

b. $N = 200$, $p = .5$
Even more like normal distribution

c. As the probability of success decreases, the curve shifts to the level indicating that the probability of success has decreased