

CHI SQUARE

$$\chi^2$$

GOODNESS OF FIT TEST

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Example: Is the die fair?

Suppose someone hands you a single die. It looks normal but your friend thinks it may be loaded in such a way that it favors a "6" when rolled. Your friend would like you, as a scientist, to find out if it is loaded or fair.

One thing you could do is do an experiment. You could roll it 50 times and count the frequency of 6's, 5's, 4's, 3's, 2's, and 1's you get. That should tell you something. If each of the numbers (from 1 to 6) shows up with about equal frequency, then the die appears fair. If 6 shows much more frequently than the other numbers, then the die appears loaded.

Let's say:

Scientific Hyp: The die is ...

Skeptical Hyp: The die is ...

Do an experiment:

Roll the die 60 times

Results:

To make a VALID argument for the Sci Hyp over the Skeptical Hyp at the LEVEL OF DATA you must first get a pattern of results (data) that are predicted by the scientific hyp.

Do the results above fit the sci hyp?

If you get results favoring the scientific hypothesis (very frequent 6's), there is the PCH of CHANCE to deal with:

You are rolling a die. The results (frequent 6's) could have happened by chance alone.

Chi-square Goodness of fit test evaluates the PCH of chance.

[Of course, you also have to establish validity at the level of design so that you have internal, construct, and external validity too. The Chi-square test only deals with statistical conclusion validity]

Criteria for using Chi-square:

1. Partition...

Categories can be nominal ...

2. Prior Probabilities...

Prior prob for j th category = P_j

3. N independent observations...

4. Frequency data...

Example: Roll a die

Fits each of the four above criteria:

Partition:

Prior Probabilities

N independent trials

Frequency data

Expected frequency in each category:

$$E(f_j) = fe_j =$$

Thus for every category, you now have an

Observed frequency in each category: (data)

$$f_{o_j} =$$

Statistical Hypotheses

$$H_0:$$

$$H_1:$$

FORMULA FOR CHI-SQUARE

$$\chi^2 = \text{CHI SQUARE} =$$

$$df =$$

RATIONALE FOR FORMULA (Optional)

$$(f_{o_j} - f_{e_j}) \text{ gives us}$$

$(fo_j - fe_j)^2$ gives us

dividing by fe_j gives us

summing gives us

Final rationale

Large values of CHI-SQUARE mean

Low values of CHI-SQUARE mean

EXAMPLE OF COMPUTATIONS

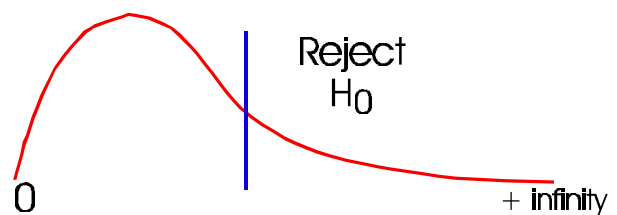
PUTTING REJECT H_0 REGIONS ON THE RANGE OF THE TEST STATISTIC

Looking up critical values



Sampling Distribution of Chi Square

1)
Assumptions about
where data come from



4) Sampling Distribution of Ch Square

2) Sample: Frequency Data

3)

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Sample Statistic: